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ABSTRACT

Six Piagetian-type tasks, designed to examine associated topological groupings, were examined to determine if a hierarchical relationship existed among them. The groupings examined were: the partitioning of sets and addition of subsets, reciprocity of proximities, one-to-one multiplication of elements, order of placement, symmetrical interval relations, and one-to-one multiplication of relations. The tasks were individually administered, in random order, to thirty-six students from each grade one, three, and five. The order of attainment of the tasks indicated by this study differs from the order suggested by Piaget and Inhelder. The author presents reasons for this difference, and implications of the study useful to educators are discussed. (MK)

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The Scaling of
Six Topological Groupings

by

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SE 030 106

Problem Statement

The primary purpose of this investigation was to examine whether a hierarchical relationship existed among six Piagetian-type tasks designed to examine associated topological groupings.

Rationale for the Study

Many science curricula have been developed during the last decade and a half. These curricula have utilized various theoretical bases. The theory stated to be the foundation upon which many of these curricula were based is Piaget's theory of cognitive development. To one extent or another these curricula seem to have disregarded some of Piaget's findings and some of the pedagogical guidelines which might be culled from these findings. (Readers interested in pursuing some of these guidelines are directed to an interview Piaget had with Eleanor Duckworth which appeared in Learning, Oct. 1973, Vol. 2, No. 2, pp. 22-29).

One reason which might underlie the above mentioned omission is the lack of clarification and elaboration of the 'fine-structure' of many of the components of Piaget's theory. The fulfillment of this omission would have important implications for curriculum development. Piaget and Inhelder (1948) recognized this need when they suggested that the clarification and elaboration of spatial conceptualizations would suggest how all areas of intellectual development proceed. Also, the more fully we comprehend how a child conceptualizes space the more likely we will be able to understand how he interprets his physical environment. Having knowledge of this latter aspect would enable educators to better facilitate the acquisition of knowledge in general, and spatial knowledge in particular. That clarification of this sort has not occured in other areas of intellectual development as well as in the realm of the development of spatial

concepts has been noted by others. (Belanger, 1969; Martin, 1976). As a result of this situation what seems to have happened is that oftentimes educators and curriculum designers while filling in and interpreting Piaget's theory make errors which could have serious consequences when incorporated into curricula.

This study was undertaken in order to elaborate the fine-structure' of the topological groupings as proposed by Piaget and Inhelder (1948). Specifically, the study was conducted in order to help resolve a conflict between the hierarchical relationship among the topological groupings reported by Cohen (in press) and the order Piaget and Inhelder (1948) hypothesized to exist amongst these groupings.

The following is offered as a brief description of the topological groupings examined in this study.

Grouping I - The partition of sets and addition of subsets: This grouping involves operations dealing with breaking up of a continuous whole into a neighboring elements and bringing them together again to form a continuous whole.

Grouping II - Reciprocity of proximities: This grouping allows the child to realize that if he begins with the element A and regards A' as its neighbor (expressed in the form A_1 and A_1') he can also mentally travel in the other direction and begin with ^{1}A' and ^{1}now regard A as its neighbor.

Grouping IV - One-to-one multiplication of elements: This grouping deals with operations which enable a child to understand dimensions as topological enclosure relations. For example, an object inside of a box can only be linked with another outside by passing through one side of the box; that is, be intersecting a surface.

Grouping V - Order of placement: This grouping allows an individual to understand both the direct and the reverse sequence of a group of elements.

Grouping VI - Symmetrical interval relations: This grouping permits an individual to understand the relationship of 'between'. For example, B is between A and C in direct order, just as it is between C and A in reverse order.

Grouping VIII - One-to-one multiplication of relations: This grouping permits an individual to form relationships between two-or-three dimensional networks. These take the form of one-to-one correspondences which the child establishes between pairs of ordered sets.

Piaget (1948) suggests that groupings develop in a hierarchical manner,

each grouping being derived from other groupings. That this hierarchical-type of development does occur, to one degree or another, has been substantiated by other researchers. (Dettrick, 1974; Camp, 1975; Odegaard, 1975; Reesink, 1976; and Doyle, 1977).

A study concerning the 'fine-structure' of topological space seems to be the appropriate point to begin since the other spatial conceptualizations, namely those of a projective and of a Euclidean nature, are derived from the topological conceptualizations. (Piaget and Inhelder, 1948; Laurendeau and Pinard, 1970). If topological concepts of space develop independent of and precede projective and Euclidean concepts, then in order to have complete understanding of these later developing concepts and in order to promote their development, one must first elaborate the development of the topological groupings.

The following are offered as examples of instances of where the ability to spatially conceptualize fully is necessary for comprehension of certain science concepts. In biology, the use of cross- and longitudinal-sections; in geology, the understanding of crystal structure, and in chemistry the understanding of atomic and molecular structure.

Furthermore, these same abilities are necessary in order for a student to be able to imagine or reason using relationships involving three-dimensions formed from a two-dimensional presentation and vice versa. Such transformations are often assumed when scientific concepts are presented visually.

Knowledge of the order of hierarchical development of the toplogical groupings would permit educators to make available activities in a sequential manner which would permit students to structure their thinking more effectively.

Procedure

The sample for this study was drawn from six elementary school populations: Three from Tempe, Arizona and three from the Madison School District in Pheonix,

Arizona. The sample from this study consisted of students in first, third and fifth grades, their respective average ages at the time of interviewing being 6:6, 8:7, and 10:9. Thirty-six subjects, six from each school sampled were randomly selected for each of the three grade levels, yielding a total N of 108.

Six Piagetian-type tasks were administered in a ramdonized order to each of the subjects individually by this researcher. These tasks were derived from the grouping descriptions provided by Piaget and others. The following abbreviations will be used in describing each grouping: I = infralogical and To = topological. In this study the groupings were numbered so that they corresponded to their homologues in the logical domain. The groupings examined in this study were: ITo_1 (Partition of Sets and Addition of Subsets); ITo_2 (Reciprocity of Proximities); ITo_4 (One-to-One Multiplication of Elements); ITo_5 (Order of Placement); ITo_6 (Symmetrical Interval Relations); and, ITo_8 (One-to-One Multiplication of Relations).

Each task, with the exception of the task used to examine grouping ITo_4, which was designed specifically for this research by this investigator, were designed on the basis of descriptions provided by Piaget (1965), Piaget and Inhelder (1948) and Piaget and Szeminska (1960). These descriptions were incomplete, and therefore the tasks are different from tasks utilized by other researchers. Each one of the tasks was designed to determine whether a subject possessed the cognitive ability described by a particular grouping. If a subject successfully completed all task parts then it was inferred that the subject possessed that grouping. The tasks were administered in a ramdomized order. Each task was scored on the basis of pre-determined criteria derived from the literature and the results of a prior study (Cohen, in press). In order to provide a reliability check of the scoring of the tasks, a second person who

was knowledgeable about the work of Piaget, with previous experience in scoring and evaluation of interviews, scored a randomized sample of the data to assure consistency in the scoring of task responses on the part of the investigator. Also, another individual, who due to his professional involvement with Piagetian-type research, was considered an expert responded that the tasks appeared to be measures of the groupings they were designed to assess.

Each one of these tasks permitted the subject being interviewed to interact with objects (either overtly or representationally) and to respond to questions regarding his actions upon the objects. Most of the protocols required that the subject supply an acceptable reason for a choice.

Results

For the purposes of the analysis used in this study only those subjects who completed each part of a task without any errors were given a "pass". Otherwise, the subject was given a "fail" for that task. The subject's responses to each one of these tasks were categorized utilizing a set of criteria established by this researcher, and based on descriptions provided by Piaget (1965), Piaget and Inhelder (1948) and Piaget and Szeminska (1960). A Chilton modified Guttman Scalogram Analysis (Edwards, 1957) was utilized to test whether the scores from the Piagetian-type tasks formed a unidimensional scale, a unidimensional scale being a set of items that fall along a single continuum. Within this continuum the items are ordered according to their difficulty. However, the distance between the items with regard to difficulty is not indicated. In the case of this study the question was whether the pattern of passes and fails of the predicted order of topological groupings forms a gradation that can be considered a unique developmental trend.

Table 1 provides the number of subjects who passed each of the six tasks.

TABLE 1

Number and Percent of
Subjects Passing Each Task

Grade Level	N	ITo_1	ITo_2	ITo_4	ITo_5	ITo_6	ITo_8
1	36	6	5	1	17	2	15
3	36	18	20	4	32	19	32
5	36	28	33	17	34	30	35
TOTAL	108	52	58	22	83	51	82
Percent of Total N Passing		48	54	20	77	47	76

The null hypothesis that there is no difference in the observed order of occurrence of the scores obtained from the Piagetian-type tasks used and the order which would have occurred by chance was tested.

A modified Guttman Scalogram, as suggested by Chilton (1966) was applied to the scores obtained for the topological groupings 1, 2, 4, 5, 6, and 8.

A Z-score of - 2.33 was obtained when the data from the above six tasks were analyzed by Chilton's technique. The null hypothesis was rejected for the task scores of ITo_1; ITo_2; ITo_4; ITo_5; ITo_6; and ITo_8 since the Z-score exceeded the critical Z-score at the .05 level. The order of attainment in order of increasing difficulty was found to be: ITo_5; ITo_8; ITo_2; ITo_1; ITo_6; and ITo_4.

Discussion

The order of attainment of the topological tasks as indicated by this study is identical to the order of attainment in an earlier study (Cohen in press).

Both studies then yielded identical order of attainments for the same topological tasks: ITo_5; ITo_8; ITo_2; ITo_1; ITo_6; and ITo_4.

The order of attainment as indicated by these studies is not congruent with the order of attainment suggested by Piaget and Inhelder (1948) in their discussion of the topological groupings. Their suggested order of attainment as understood by this investigator is ITo_1; ITo_2; ITo_5; ITo_6; ITo_4; and ITo_8.

The difference between the obtained results and Piaget and Inhelder's suggested order of attainment of the topological groupings could be due to a variety of factors, either individually or in combination. Five possible factors were proposed earlier (Cohen, in press). They will only be briefly listed here:

1. Inappropriate tasks to assess possession of groupings.
2. Mode of administration of the tasks.
3. Population differences.
4. Time difference between when Piaget and Inhelder examined their subjects and the point in time the subjects for these studies were interviewed.
5. Differences in sample size.

Other possibilities exist for the difference. One might be misinterpretation of Piaget and Inhelder's proposed order of attainment. A second possibility has to do with the methods of analysis employed to ascertain the order of attainment.

Lastly, Piaget and Inhelder (1948) base their proposed order on a detailed psychological analysis of the various components they suggest underlie the groupings. Their results were reported in a descriptive manner. No precise information was given with respect to sample size, sample compostiion, nor to individual subject performance. At no point in their analysis do they offer any statistical support for their hypothesized order of attainment.

Thus, it is suggested that the findings reached in this study and the original present a more accurate picture of the sequence of attainment of the topological groupings than does Piaget's and Inhelder's proposed sequence.

Implications

Knowledge of the order of development would permit educators to make available activities in a sequential manner which would permit students to structure their thinking more effectively. Doing this should facilitate the realization of higher operational thought processes for a larger segment of the population than is now realized.

For example, in the realm of topological conceptualizations, since grouping ITo_5, Order of Placement, develops first, teachers should provide opportunities for children to work with materials in which the children have to deal with these ideas. It is only through encountering situations entailing specific problems that a child will be able to develop the appropriate thinking processes. This is not meant to suggest that only materials enhancing the development of one grouping be provided at a time. It is likely that all of the topological groupings are being structured somewhat simultaneously. However, if the teacher is knowledgeable of which abilities might appear prior to others, then the teacher would know how to effectively proceed. That is, which materials to make available

and what alternative activities to suggest to the child to motivate the child to interact with the materials.

If children completely developed topological conceptualizations then they would have the pre-requisite structures upon which to develop projective and Euclidean conceptualizations.

Development of spatial conceptualizations might be maximized by providing children close physical experience with a variety of objects. Since so many children in our society appear to spend less and less time with physical objects in today's world educators would do well to incorporate more "hands-on" activities into their daily lessons. These activities should require the student to exercise his mental spatial abilities, and thus provide the child with the opportunity for structuring relationships.

In order to facilitate the above type of "learning" a teacher could use either readily available materials, or commercially prepared materials such as are available from Elementary Science Studies, Science Curriculum Study, Science-A Process Approach, and others. Working with printing materials, clay, drawing, and specific units such as Mirror Cards, Tangrams, Pattern Blocks, and Geo Blocks from Elementary Science Study are just a few examples of activities whose usage would facilitate the development of spatial conceptualizations, many of which are necessary for fuller understandings in science.

REFERENCES

Belanger, M. Learning studies in science education. Review of Educational Research, 1969, 39, p. 377 - 395.

Camp, D. An investigation of six logical groupings of concrete operational thought. Unpublished doctoral dissertation. The University of Iowa, 1975.

Cohen, H. The scaling of six topological Piagetian groupings as well as the affect that certain selected variables have on the attainment of these groupings and some of their homelogues in the logical domain. Journal of Research in Science Education, in press.

Dettrick, G. A study of the structure of Piagetian logical and infralogical groupings within the concrete operational period of cognitive development. Unpublished doctoral dissertation, The University of Iowa, 1974.

Doyle, J. An investigation of Piaget's infralogical groupings dealing with projective space. The University of Iowa, 1977.

Duckworth, E. Piaget takes a teacher's look. Learning the Magazine for Creative Teaching, Oct. 1973, Vol. 2, No. 2, 22-27.

Edwards, A. Techniques of attitude scale construction. New York: Appleton - Century-Crofts, (chp. 7), 1957.

Laurendeau, M. and Pinard, A. The development of the concept of space in the child. New York: International Universities Press, Inc. 1970.

Martin, J., A test with selected topological properties of Piaget's hypothesis concerning the spatial representation of the young child. Journal of Research in Mathematical Education, 1976, 7, 26-38.

Odegaard, G. An investigation of Piaget's groupings: Seriation and projective space. Unpublished doctoral dissertation, The University of Iowa, 1975.

Piaget, J. The child's conception of number. New York: Norton Library, 1965.

Piaget, J. Intellectual evolution from adolescence to adulthood. Human Development. 15 (1): 1 - 12. 1972.

Piaget, J. and Inhelder, B. The child's conception of space, New York: Norton Library, 1948.

Piaget, J. and Szeminska, A. The child's conception of geometry, New York: Basic Books, 1960.

Reesink, C. The performance of secondary school students on Piaget-type tasks concerning projective and Euclidean space. Unpublished doctoral dissertation, The University of Iowa, 1976.